



## An Improved Model for Prediction of the Effective Thermal Conductivity and viscosity of TiO<sub>2</sub>-water Nano-fluid

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**Abstract:** Thermal conductivity is an important characteristic of a nano-fluid. This paper presents models for the prediction of the effective thermal conductivity and viscosity of titanium oxide based on water by used of dimensionless groups. The models express the thermal conductivity of a nano-fluid as a function of the thermal conductivity of the interfacial shell and volume fraction and the viscosity of nano-fluid as function of the temperature and volume fraction. The model of effective thermal conductivity includes four regions that by analysis of presents models for the regions, can be obtained effective value of dependence parameter. The model shows that for volume fraction less than 1% and diameters less than 20 nm intensity of increase thermal conductivity is much more than other region. Know, with decrease of concentration, the viscosity of nano-fluid decreased, so, this region, is the best region for applications of heat transfer devices, because, pressure drop also decreased.

**Keywords:** Critical particle size, nano-fluids, thermal conductivity and dimensionless groups.

### الگوی جدید برای پیش‌بینی هدایت حرارتی موثر و لزجت نانوسیال آب - TiO<sub>2</sub>

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**چکیده:** هدایت حرارتی از خواص مهم نانوسیال‌ها می‌باشد. این مقاله، مدل‌هایی را برای پیش‌بینی هدایت حرارتی موثر و لزجت نانوسیال آب - اکسیدتیتانیوم (TiO<sub>2</sub>) براساس گروه‌های بی‌بعد معرفی می‌نماید. این مدل‌ها، هدایت حرارتی نانوسیال‌ها را بصورت تابعی برحسب ضریب هدایت حرارتی در فصل مشترک سیال پایه و ذرات فلزی و کسر حجمی و ضریب هدایت حرارتی نانوسیال را برحسب دما و کسر حجمی بیان می‌کنند. مدل هدایت حرارتی موثر نانوسیال شامل چهار ناحیه می‌باشد که با تحلیل مدل‌های حاضر برای این چهار ناحیه می‌توان مقدار موثر پارامتر را تعیین نمود. نتایج این مدل نشان می‌دهد که برای کسر حجمی کوچکتر از ۱٪ و برای قطر ذرات فلزی کوچکتر از ۲۰ نانومتر، شدت تغییرات هدایت حرارتی بیشتر از حالت‌های دیگر می‌باشد. با آگاهی از این موضوع که با کاهش غلظت، لزجت نانوسیال کاهش می‌یابد، این ناحیه، ناحیه بهینه برای کاربرد نانوسیال بعنوان سیال انتقال حرارت می‌باشد زیرا علاوه براین افت فشار نیز کاهش می‌یابد.

**واژه‌های کلیدی:** اندازه بحرانی ذرات، نانوسیال، ضریب هدایت حرارتی و گروه بی‌بعد.

## 1. Introduction

Heat transfer plays an important role in numerous applications where the thermal energy transfer between fluid through some heat transfer devices. Increasing the heat transfer efficiency of these devices is desirable, because by increasing efficiency, the space occupied by the device can be minimized, which is important for applications with compactness requirements.

There are several methods to improve the heat transfer efficiency. Some methods are utilization of extended surfaces, application of vibration to the heat transfer surfaces, and using of micro channels which came an increase in heat transfer area and therefore increasing the size of device. In the recent decade some researches purposed heat transfer efficiency can also be improved by increasing the thermal conductivity of the working fluid and shown that the Nano-fluids have great potential for heat transfer enhancement [13, 14]. Commonly used heat transfer fluids such as water, ethylene glycol, and engine oil have relatively low thermal conductivities, when compared to the thermal conductivity of solids. High thermal conductivity of solids can be used to increase the thermal conductivity of a fluid by adding solid particles in nano size to that fluid.

Experimental studies show that thermal conductivity of Nano-fluids depends on many factors such as particle volume fraction [2,5]. Particle material[25,26], particle size[25,28], particle shape[10,15,29], base fluid material[8,27], and temperature[2,11,14]. But In order to justify of anomalous thermal conductivity enhancement of Nano-fluid, some researches have explained several reasons And models based on those presented such as; Brownian motion of nano-particles [1,7,8], clustering of nano-particles [9,16,17,21,22], liquid laying around nano-particles [3,4,6], and ballistic phonon transport in nano-particles

[18,19].

Another important factor in flow of canal is the viscosity, so that with increases of the viscosity, power of pumping increased and system efficiency decreases. Therefore, for using of Nano-fluids in practical applications, the rate of viscosity increase of Nano-fluids with respect to pure fluids should be thoroughly investigated. It is known that Nano-fluid viscosity depends on many parameters such as; particle volume fraction, particle size, temperature, and rate of clustering. Increasing particle volume fraction increases viscosity and this was validated by many studies [14, 15, 22]. Prasher et al. [23] indicated that Nano-fluid viscosity does not change significantly with particle size. but, Nguyen et al. [21] observed increasing viscosity with increasing particle size whereas Pastoriza-Gallego et al. [24] reported decreasing viscosity with increasing particle size. Nguyen et al[21]. also analyzed the effect of temperature on viscosity and observed a decrease in viscosity with increasing temperature. Turgut et al.[14], showed that for Tio<sub>2</sub>-Water Nano-fluid there is no dependence related to temperature and the thermal conductivity increased by the same order of magnitude as the base fluid .

In this paper, we studied both the thermal conductivity and viscosity of Tio<sub>2</sub>-Water Nano-fluid and presented models for predicting of effective thermal conductivity and effective viscosity based on dimensional analysis. And with separated areas based on volume fraction amount and particle diameter size, effective value of properties of Tio<sub>2</sub>-Water Nano-fluid discusses.

## 2. Formulation of the thermal Conductivity Model

In this section by applying the theory of dimensional analysis to the heat transfer phenomena of a nano-fluid, a new model to

estimate the effective thermal conductivity of nano-fluids will be derived. For obtaining a relatively accurate relation between thermal conductivity and other effective thermo-physical properties of a nano-fluid, this process should be analyzed with a specific nano-fluid under some pre evaluated conditions. In the present work the formulation of the proposed model for the effective conductivity is derived by the several experimental data [2, 11, 14,15,30] for TiO<sub>2</sub>-water based on the technique of dimensional analysis. By assuming the  $k_{eff}$ , the effective conductivity of nano-fluid is a function of the thermal conductivities of the base fluid ( $k_f$ ), the solid particle ( $k_p$ ), the interfacial shell ( $k_i$ ), the particle diameter ( $d_p$ ), the volume fraction of the particle ( $\phi$ ), the interfacial shell thickness ( $t$ ), the temperature of Nano-fluid ( $T$ ), and half of the base fluid boiling temperature ( $T_c$ ), it can be written as:

$$k_{eff} = f(k_f, k_p, k_i, d_p, \phi, T, T_c) \tag{1}$$

By using the theorem of dimensional analysis, the Eq.(1) is expressed in a modified form in terms of dimensionless variables as follow:

$$\frac{k_{eff}}{k_f} = f_1\left(\frac{k_i}{k_f}, \frac{t}{d_p}, \phi, \frac{T}{T_c}\right) \tag{2}$$

Usually for a nano-fluid,  $k_{eff} > k_f$  which yields:

$$\left(\frac{k_{eff}}{k_f}\right) > 1 \Rightarrow \left(\frac{k_{eff}}{k_f}\right) = 1 + R \tag{3}$$

Where R is the enhancement factors and expressed as follows:

$$R = \left[\left(\frac{k_i}{k_f}\right)^a \left(\frac{t}{d_p}\right)^b \phi^c \left(\frac{T}{T_c}\right)^d\right] \tag{4}$$

Combining Eqs. (8) and (9), it yields a

general form for the relative effective thermal conductivity as follows:

$$\frac{k_{eff}}{k_f} = 1 + \left[\left(\frac{k_i}{k_f}\right)^a \left(\frac{t}{d_p}\right)^b \phi^c \left(\frac{T}{T_c}\right)^d\right] \tag{5}$$

Where the thickness of which,  $t$ , can be calculated from the well-known Langmuir formula:

$$t = \frac{1}{\sqrt{3}} \left(\frac{4M_f}{\rho_f N_A}\right)^{1/3} \tag{6}$$

where  $M_f$  is the molar mass of the liquid,  $\rho_f$  is the density of the liquid, and  $NA$  is Avogadro's number which equals  $6.023 \times 10^{23}$  mol. The thickness of the adsorption layer,  $t$ , is determined by the property of the liquid, but the densification of the adsorption layer affected further the compatibility of the particle surface to the surrounding liquid. for this study Nano-fluid value of the  $t$  is 2.85 nm.

$k_i$  calculated from huaqing xie et al.[3] relation as follow

$$k_i = \frac{k_f M^2}{(M - \gamma) \ln(1 + M) + \gamma M} \tag{7}$$

With  $M = \epsilon_p (1 + \gamma) - 1$ , where  $\epsilon_p = k_p / k_f$  is the reduced thermal conductivity of nano-particle and  $\gamma = \delta / r_p$  is ratio.  $k_p$  value is 11.7 (w/mK) of the nano-layer thickness to the original particle radius.

It should be noted that for a given thermal conductivity of nano-fluid, thermal conductivity of intermediate layer according of the Eq.(7) will be only a function of particle diameter. Turgut et al.[14], showed that for TiO<sub>2</sub>-Water nano-fluid there is no dependence related to temperature and the thermal conductivity increased by the same order of magnitude as the base fluid which is water

(Eq.(8)). then can be written the value of  $k_f$  from Eq.(5) is zero.

$$k_f = -1.13 + 0.00975 T - 0.000131 T^2 \quad (8)$$

T is the temperature of Nano-fluid to °K. Then Eq.(5) simplifies into:

$$\frac{k_{eff}}{k_f} = 1 + \left[ \left( \frac{k_i}{k_f} \right)^a \left( \frac{t}{d_p} \right)^b \varphi^c \right] \quad (9)$$

Now with use the experimental data [2, 11, 14, 15,30], can be conclusion value of a, b, c, d from Eq.(5) by applying the total least squares method. But with analyzed the data and the answers can be calculated that should be the study break up to four regions as follow:

### 3. Formulation of the Viscosity Model

In this section by applying the theory of dimensional analysis to the heat transfer phenomena of a nano-fluid, a new model to estimate the effective viscosity of nano-fluids will be derived. For obtaining a relatively accurate relation between viscosity and other effective thermo-physical properties of a nano-fluid, this process should be analyzed with a specific nano-fluid under some pre evaluated conditions. In the present work the formulation of the proposed model for the effective viscosity in derived by the several experimental data [14] for TiO<sub>2</sub>-water based on the technique of dimensional analysis. By assuming the  $\mu_{eff}$ , the effective viscosity of nano-fluid is a function of the viscosity of the base fluid ( $\mu_f$ ), the volume fraction of the particle ( $\varphi$ ), the temperature of nano-fluid (T), and half of the base fluid boiling temperature ( $T_c$ ), it can be written as:

$$\mu_{eff} = f(\mu_f, \varphi, T, T_c) \quad (10)$$

By applying of dimensional analysis, the Eq.(10) is expressed in a modified form in

terms of dimensionless variables as follow:

$$\frac{\mu_{eff}}{\mu_f} = f_2 \left( \varphi, \frac{T}{T_c} \right) \quad (11)$$

By analyzing the behavior of effective viscosity into temperature and volume fraction of nano-fluid using experimental data [14]. It yields a general form for the relative effective thermal conductivity as follows:

$$\mu_{eff} = \frac{1 + 1500 \varphi^2}{\left( \frac{T}{T_c} + 0.6 \right)^{1.29}} \quad (12)$$

T and  $T_c$  calculated with °C and  $\mu_{eff}$  obtained to mpa.s.

## 4. Results and Discussions

The models for prediction of effective thermal conductivity and the effective viscosity of nano-fluid which discussed in previous section, applied and tested in TiO<sub>2</sub>-water systems. The models parameters have been defined for these systems and results are summarized in Table II and Eq.(12) for thermal conductivity and viscosity, respectively : The models validation and its accuracy were performed by enormous experimental data published in literature and outperform previously derived models [2, 11, 14,15,30]. The comparative results of the model predictions are shown in Fig.1-a to Fig.1-g and Fig.2 for TiO<sub>2</sub>-water system. Careful examinations of the figures it reveals that the model is in good agreement with experimental data and outperforms previously derived models when applied to examined Nano-fluids.

In order to study behavior of effective thermal conductivity of TiO<sub>2</sub>-Water nan-ofluid in the obtained equation should be found gradient of effective thermal conductivity to volume fraction and interfacial shell thermal conductivity.

Compared with the ranges ( $d_p < 20$ ,  $\phi < 1\%$ ) and ( $d_p \geq 20$ ,  $\phi > 1\%$ ) can be seen, the average gradient of the relative of effective thermal conductivity to the volume fraction for smaller particles is more than the large particles (9 times). This could be due to two factors, first, in the same volume fractions, the smaller particles have more effective area. And the total volume of the intermediate layer formed around the particles for the smaller particles more than the large particles with the same volume fraction. As the gradient of the effective thermal conductivity to  $(t/d_p)$  can be seen, for smaller particles, this gradient is much higher (about 13 times). Also with note that to gradient of the effective thermal conductivity to  $(k_i/k_f)$  can be seen, effect of intermediate layer for smaller particle 17 times more than large particle.

Compared with the ranges ( $d_p < 20$ ,  $\phi < 1$ ) and ( $d_p < 20$ ,  $\phi \geq 1$ ) can be seen that the increase in thermal conductivity compared to the volume fraction, for high volume fractions of a percent, the volume fraction of a percentage is much lower (about 7 times). This phenomenon can be likened to being a aggregation, because with increasing volume fraction of particle accumulation and deposition may be more. Also, the gradient of the effective thermal conductivity, thermal conductivity of the interfacial layer is seen to increase for the higher volume fractions is higher (1.5 times). Given the small increase, it can be concluded that the intermediate layer increases with increasing volume fraction of the total amount. But the phenomenon of being a aggregation, this volume also will reduce the total result of the increase will average 1.5 percent.

Compared with the ranges ( $d_p \geq 20$ ,  $\phi < 1$ ) and ( $d_p \geq 20$ ,  $\phi \geq 1$ ) can be seen that gradient of the effective thermal conductivity to volume fraction for ( $\phi \geq 1$ ) little more than when ( $\phi < 1$ ) because with increased volume fraction

amount of clustering increases and with note that gradient of effective thermal conductivity to  $(t/d)$ ,  $(K_i/k_f)$  is negative, then can be conclusion that increase amount of particles, regardless of particle size increased. The volume of intermediate layer decreased, then the gradient of the effective thermal conductivity clustering and also with increases of particle size total gradient than in both the volume fraction almost is equal. The phenomenon to  $(k_i/k_f)$  will be negative

## 5. Conclusions

In this paper, we developed an empirical model for prediction of the effective thermal conductivity of  $\text{TiO}_2$ -Water nano-fluids, based on the dimensionless variables of  $(k_i/k_f)$ ,  $\phi$ ,  $(t/d_p)$ , and  $(T/T_c)$ . The proposed model accounts for the interfacial shell, and aggregation of particle. The validation of the model is verified by applying the results obtained by the experiments of  $\text{TiO}_2$ -water systems. The models parameters have been calculated by applying it to data of systems obtained by experiments and published in literature.

The model of effective thermal conductivity includes four regions that by analysis of presents models for the regions, can be obtained effective value of dependence parameter. The model shows that for volume fraction less than 1% and diameters less than 20 nm intensity of increase thermal conductivity is much more than other region. And with analyzes of the equations Can be conclude that enhancement thermal conductivity depend on liquid layering around nano-particle and clustering or aggregation of particle. and with obtained the gradient of variation relative effective thermal conductivity with volume fraction and relative interfacial shell thermal conductivity can be conclude that effect of aggregation for volume fraction above 1% and diameter under 20nm more than other region and effect of liquid

layering for particle diameter under 20nm much more than to above 20nm. Also we developed an empirical model for prediction of the effective viscosity of  $\text{TiO}_2$ -Water nano-fluids, based on the dimensionless variables of  $\phi$ , and  $(T/T_c)$ . The model shows for volume fraction above 1% enhancement of viscosity much more. So clarified, that the best region for applications, it is the volume fraction under 1% and particle diameter under 20 nm.

## 5. References

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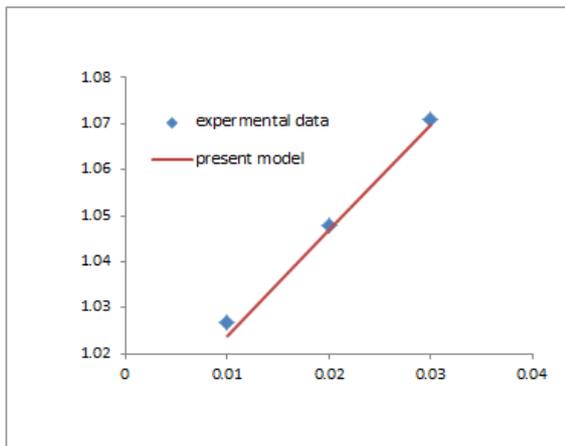
**Table (1): Constant Parameters for TiO<sub>2</sub>-Water Systems**

Constant parameter	quantity			
	$\phi > 1\%$ , dp > 20nm	$\phi \leq 1\%$ , dp > 20nm	$\phi > 1\%$ , dp < 20nm	$\phi \leq 1\%$ , dp < 20 nm
a	2.6024	3.114	2.1821	5.721
b	-0.219	-0.017	0.0125	0.3703
c	0.9752	1.0676	0.2681	1.6668

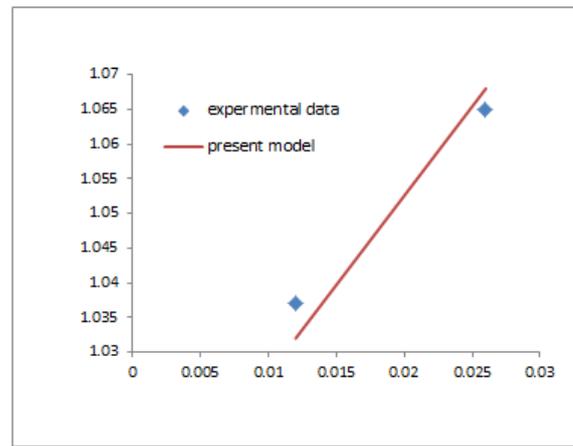
TABLE III

Table (2): Ratio gradient of relative effective thermal conductivity to volume fraction and relative interfacial shell thermal conductivity of Tio2-Water

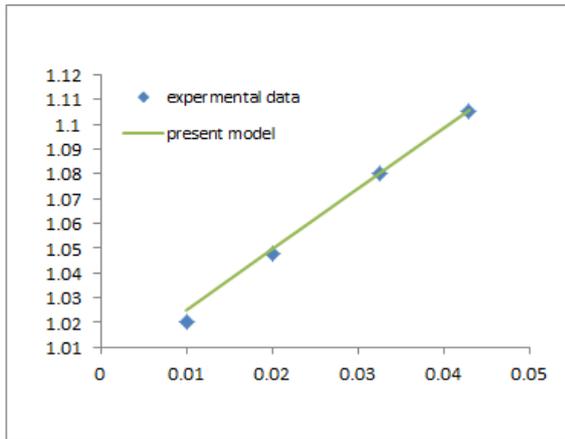
gradient Ratio	Average gradient quantity			
	$\varphi \geq 1\%$ , $d_p \geq 20\text{nm}$	$\varphi < 1\%$ , $d_p \geq 20\text{nm}$	$\varphi \geq 1\%$ , $d_p < 20\text{nm}$	$\varphi < 1\%$ , $d_p < 20\text{nm}$
$\frac{\delta(k_{eff} / k_f)}{\delta\varphi}$	2.26	2.49	3.39	23
$\frac{\delta(k_{eff} / k_f)}{\delta(t / d)}$	-0.0475	0.0173	0.329	0.2387
$\frac{\delta(k_{eff} / k_f)}{\delta(k_i / k_f)}$	-0.0102	0.0038	0.0877	0.0646



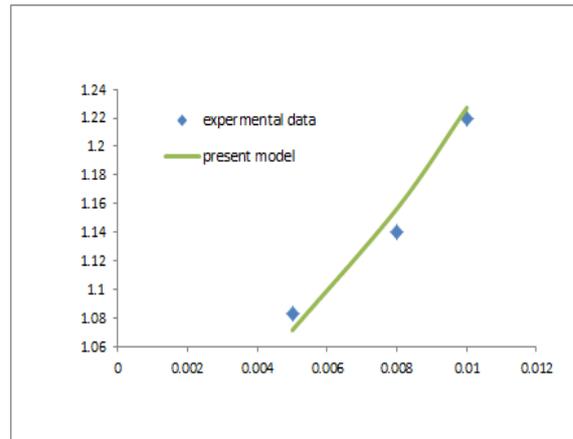
a) Particle size of  $d_p = 21\text{ nm}$



c) Particle size of  $d_p = 40\text{ nm}$



b) Particle size of  $d_p = 27\text{ nm}$



d) Particle size of  $d_p = 10\text{ nm}$

Fig. (1): Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with volume fraction ( $\varphi$ ) for  $\text{TiO}_2$  - water system and comparison of model prediction with experimental results for various particle size.

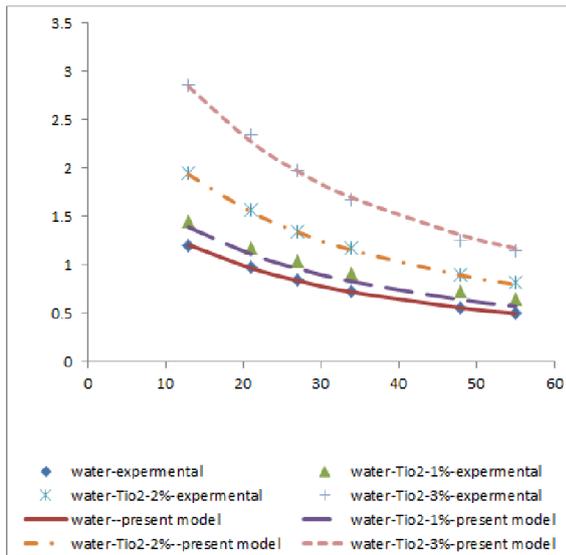
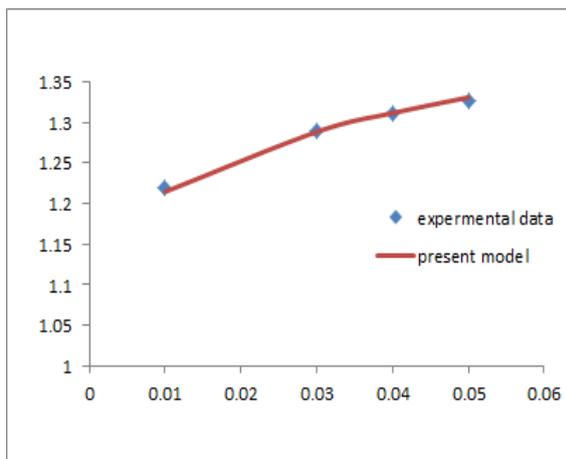
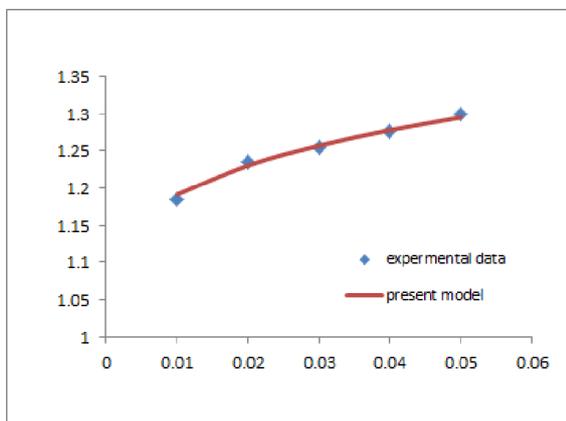


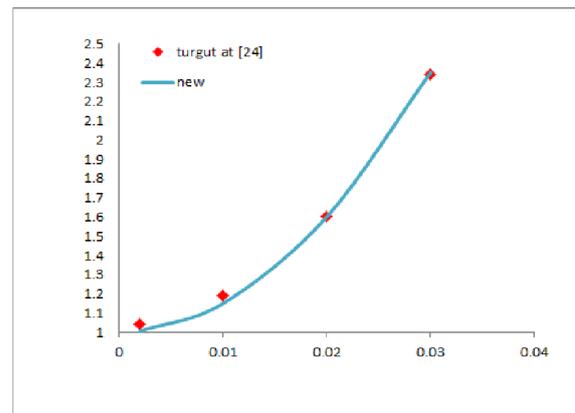
Fig. (2): Variation of relative viscosity ( $\mu_{eff}/\mu_f$ ) with temperature for  $TiO_2$ -water system for various volume fractions and its comparison with model prediction.



a) Particle size of  $d_p = 10$  nm



b) Particle size of  $d_p = 15$  nm



c) Particle size of  $d_p = 30$  nm

Fig. (3): Variation of relative viscosity ( $\mu_{eff}/\mu_f$ ) with volume fraction ( $\phi$ ) for  $TiO_2$ -water system for various particle size and its comparison with model prediction.

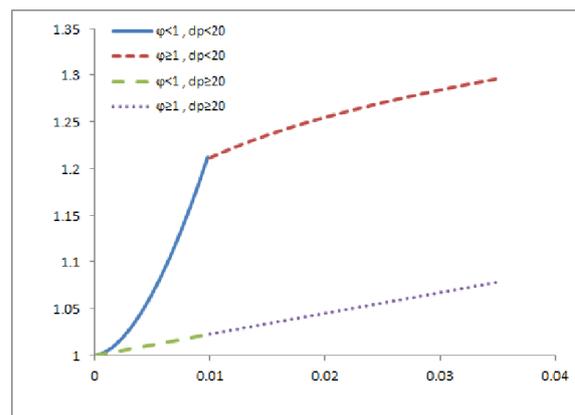


Fig. (4): Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with volume fraction ( $\phi$ )

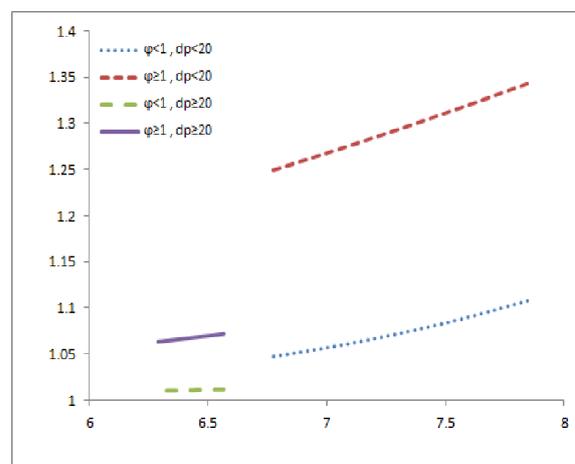


Fig. (5): Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with ( $k_f/k_f$ )

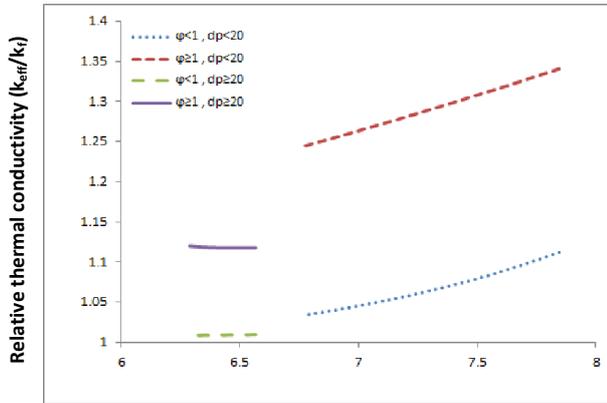


Fig. (6): Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with ( $t/d_p$ )

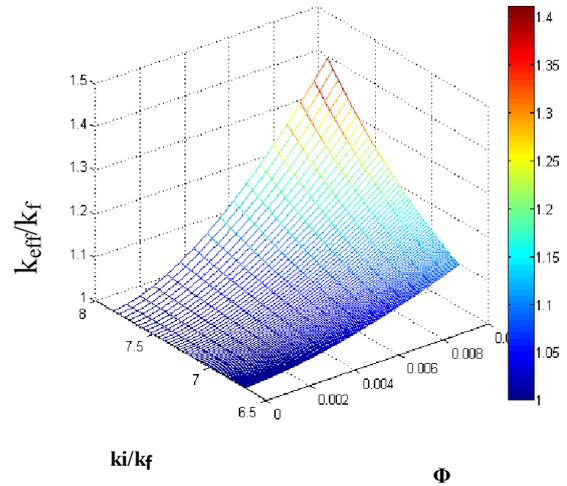


Fig.(8): Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with ( $\phi$ ) and ( $k_i/k_f$ ) for ( $\phi < 1\%$ ,  $dp < 20nm$ ) region.

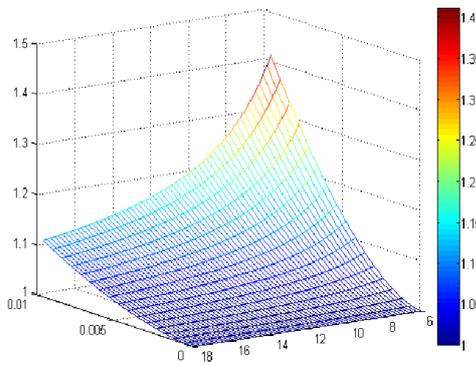


Fig. (7): Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with ( $d_p$ ) and ( $k_i/k_f$ ) for ( $\phi < 1\%$ ,  $dp < 20nm$ ) region.

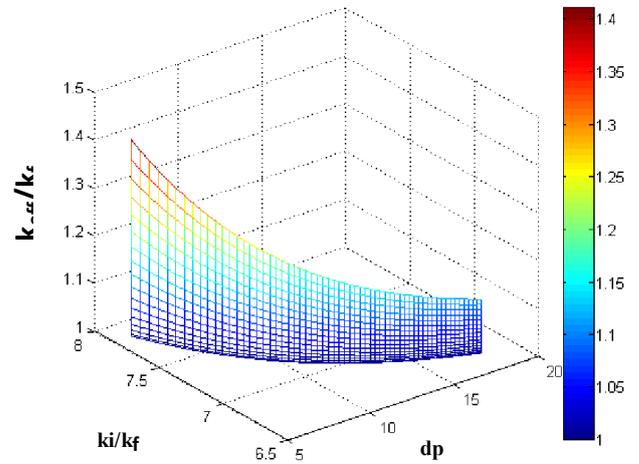


Fig. (10): Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with ( $d_p$ ) and ( $k_i/k_f$ ) for ( $\phi < 1\%$ ,  $dp < 20nm$ ) region.